UGEB2530A Game and strategic thinking
Solution to Assignment 2

Due: 9 Feb 2015 (Monday)

1. Copy the following game matrices and circle all saddle points of the matrix. **Solution:**

   (a) The game has no saddle point.

   (b) The saddle point is (1,2) which is 1.

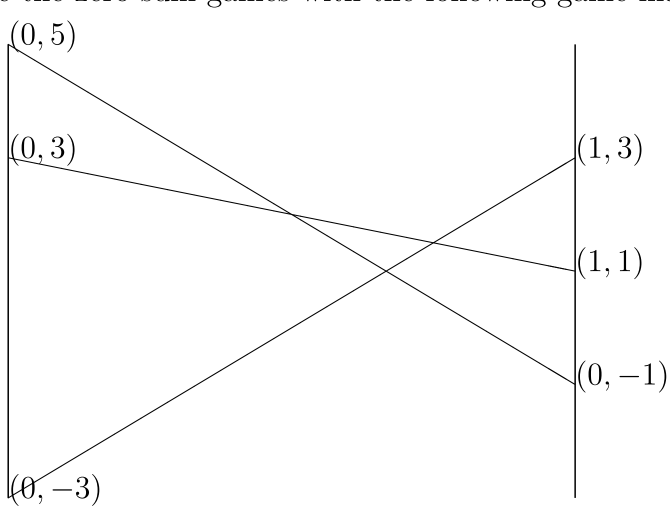
2. Solve the zero sum games, that is, find a maximin strategy for the row player, a minimax strategy for the column player and the value of the game, with the following game matrices. **Solution:**

   (a) The maximin strategy for row player:
   $p = (\frac{1-0}{3+1+1-0}, \frac{3+1}{3+1+1-0}) = (\frac{1}{5}, \frac{4}{5});$
   The minimax strategy for column player:
   $q = (\frac{1+1}{3+1+1-0}, \frac{3-0}{3+1+1-0}) = (\frac{2}{5}, \frac{3}{5});$
   The value of game:
   $v = \frac{3-0}{3+1+1-0}) = \frac{3}{5}).$

   (b) The maximin strategy for row player:
   $p = (\frac{1-4}{-2-5+1-4}, \frac{-2-5}{-2-5+1-4}) = (\frac{3}{10}, \frac{7}{10});$
   The minimax strategy for column player:
   $q = (\frac{1-5}{-2-5+1-4}, \frac{-2-4}{-2-5+1-4}) = (\frac{4}{10}, \frac{6}{10});$
   The value of game:
   $v = \frac{2-20}{-2-5+1-4}) = \frac{11}{5}).$

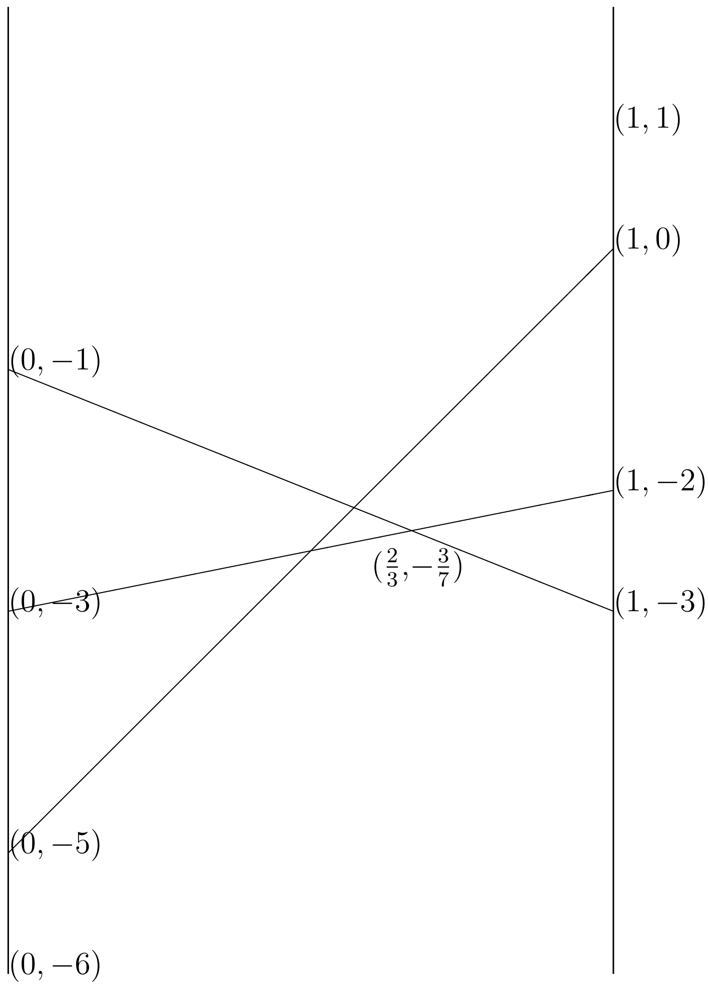
3. Solve the zero sum games with the following game matrices **Solution:**

   (a)



From the graph, it can be seen that the value of the game: $v = 1$ and $p = \frac{2}{3}$.
And if we reduce the game matrix to:
$\begin{pmatrix} -1 & 3 \\ 5 & -3 \end{pmatrix}$
We have, the the maximin strategy for row player is $(\frac{2}{3}, \frac{1}{3})$.
And $q = (0.5, 0.5)$ in the reduced game.
Therefore, the minimax strategy for column player: $(0, 0.5, 0.5)$.

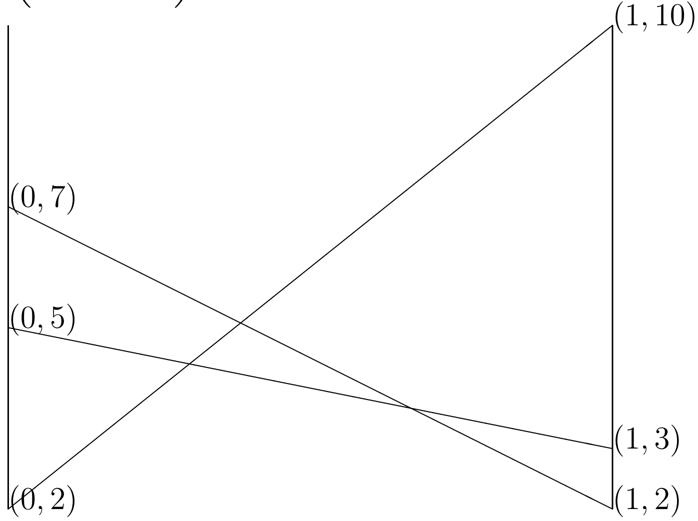
(b) First transpose the matrix negatively and we get the graph:



From the graph, it can be seen that the value of the game: $v = -\frac{3}{7}$ and $p = \frac{2}{3}$.
And if we reduce the game matrix to:
$\begin{pmatrix} -2 & -3 \\ -3 & -1 \end{pmatrix}$
We have, the the maximin strategy for row player is $p' = (\frac{2}{3}, \frac{1}{3})$.
And $q = (0.5, 0.5)$ in the reduced game.
Therefore, the minimax strategy for column player: $q' = (0, 0, \frac{2}{3}, \frac{1}{3})$. For the original game, we have:
$p = (0, 0, \frac{2}{3}, \frac{1}{3})$, $q = (\frac{2}{3}, \frac{1}{3})$.

4. Solve the zero sum game with game matrix: **Solution:**

(a) First, reduce the game to the following game matrix:
$$\begin{pmatrix} 2 & 3 & 10 \\ 7 & 5 & 2 \end{pmatrix}$$



From the graph, it can be seen that the value of the game: $v = 4.4$ and $p = 0.3$.
And if we reduce the game matrix to:
$$\begin{pmatrix} 3 & 10 \\ 5 & 2 \end{pmatrix}$$
We have, the the maximin strategy for row player is $(0.3, 0.7)$.
And $q = (0.8, 0.2)$ in the reduced game.
Therefore, for the original game, we have:
$p = (0, 0.3, 0, 0.7)$, $q = (0, 0.8, 0, 0.2)$.

5. **Solution:**

(a) $a \leq -2$.

(b) i. $v = \dfrac{6-a}{-6-a}$, therefore if $v = 0$, a has to be 6.

ii. The game matrix is:
$$\begin{pmatrix} -3 & 1 \\ 6 & -2 \end{pmatrix}$$
The maximin strategy for row player:
$p = (\frac{2}{3}, \frac{1}{3})$
The minimax strategy for column player:
$q = (\frac{1}{4}, \frac{3}{4})$